Math 55 Quiz 6 DIS 105

Name: ______________________________ 14 Mar 2022

1. A Californian license plate is made using one digit followed by three upper case English letters followed by another three digits, for example: `5FVP402` is a valid license plate.

   (a) How many license plates that contain `PIE` can be made? [3 points]

   (b) How many license plates that contain the digits `3`, `1`, and `4` can be made? [3 points]

   (c) How many license plates whose digits add up to 15 can be made? [4 points]

(a) In this case the letters are fixed already, so just considering the digits, there are $10^4 = 1000$ possible license plates.

(b) Suppose the digits are `3`, `1`, `4`, `n`.
If $n = 1, 3, 4$, then there are $4 \cdot 3 = 12$ ways to arrange these digits (or $\frac{4!}{2!1!1!} = 12$).
If $n \neq 1, 3, 4$, then there are $4 \cdot 3 \cdot 2 = 24$ (or $\frac{4!}{1!1!1!1!} = 24$).
So there are $3 \cdot 12 + 7 \cdot 24 = 204$ combinations for the digits. Together with the letters, there are $204 \cdot 26^3 = 3585204$ possible license plates.

(c) This is equivalent to asking how many solutions there are to $x_1 + x_2 + x_3 + x_4 = 15$ with each x_i an integer in $[0, 9]$.
There are $\binom{18}{3} = 816$ solutions where each x_i is a nonnegative integer, but among these, there are $\binom{8}{3} = 56$ solutions where $x_1 \geq 10$ (since solutions of this type can be thought of as solutions of $(x_1 - 10) + x_2 + x_3 + x_4 = 5$ where $x_1 - 10, x_2, x_3, x_4$ are nonnegative integers. Similarly, there are 56 solutions where $x_2 \geq 10$, 56 solutions where $x_3 \geq 10$, and 56 solutions where $x_4 \geq 10$. All of these do not overlap, so there are $816 - 4 \cdot 56 = 592$ combinations for the digits.
Together with the letters, there are $592 \cdot 26^3 = 10404992$ possible license plates.